EFFECT OF RADIAL VISCOSITY VARIATION ON NON-NEWTONIAN FLOW OF BLOOD IN AN OVERLAPPING STENOSED ARTERY

Pushkar Singh[1], Aditi Singh[2], S. P. Singh[3]
[1,2]P.G Scholar, Department of Mathematics, D.E.I., Dayalbagh, Agra (U.P)
[3]Asst. Prof., Department of Mathematics, D.E.I., Dayalbagh, Agra, (U.P)
Email: pushkar19121992@gmail.com[1], aditi.dei2014@gmail.com[2]
shail.dei@gmail.com[3]

Abstract
The proposed work involves the thorough investigation of the Non-Newtonian flow of blood through an overlapping stenosed artery. Herschel-Bulkley equation has been taken to represent the Non-Newtonian behavior of blood. The study shows the effect of radial viscosity variation on various parameters of blood flow. The expressions for flow rate, resistance to flow and wall shear stress has been obtained analytically. The variations of these flow parameters are shown with the help of graphs. It is found that various flow characteristics are affected by the presence of overlapping stenosis and radial variation of blood viscosity.
Keywords: Blood flow, Blood viscosity, Overlapping stenosis, Herschel-Bulkley fluid, Flow rate, Resistance to flow, Wall shear stress.

I. INTRODUCTION

Healthcare problems are apparently concerned by the people in these days. For over centuries, cardiovascular diseases have been noticed as one of major illnesses where numerous people suffer from them. These diseases are a major cause for deaths in this world. Among the cardiovascular disease, the familiar one is atherosclerosis. If the stenosis is present in the artery, normal blood flow is disturbed. The intimal thickening of stenosed artery was understood as an early process in the beginning of atherosclerosis. This may be caused by unhealthy living conditions such as exposure to tobacco smoke, lack of physical activity and improper dietary habits. It is always followed by the serious changes in blood flow, pressure distribution, wall shear stress and flow resistance, thus leading to the importance of study of blood flow through stenosed artery. A Newtonian fluids, by definition is one in which the coefficient of viscosity is constant at all rates of shear. However, the fluids which do not obey the linear relationship between shear stress and strain rate are called as Non-Newtonian fluids. In few studies (Young [15]), the behavior of the blood has been considered as a Newtonian fluid. However, it may be noted that the blood does not behave as a Newtonian fluid under certain conditions. It has been observed that whole blood; behave as Newtonian at high shear rate while at low shear rates and in small diameter arteries (Cokelet et. al [5], Charm and Kurland [3]), it exhibits Non-Newtonian behavior.

Further (Scott-Blair and Spanner [1]) reported that blood obeys the Casson equation only in a limited range, not at very high and very low shear rates. It is observed that the Casson fluid model can be used for moderate shear rates in smaller diameter tubes whereas the Herschel-Bulkley fluid model can be used at still lower shear rate flow in very narrow arteries where the yield stress is high. The Herschel-Bulkley equation contains one more parameter than the Casson equation does; it would be expected that more detailed information about the blood properties can be obtained by the use of the Herschel-Bulkley equation. Further, in small diameter tubes blood behaves like a Herschel-Bulkley fluid rather than Power law and Bingham fluids

(Chaturani and Samy [4]). However, all these investigations considered the effect of single stenosis but, the constrictions may develop in series (multiple stenoses) or may be of irregular shapes or overlapping. (Chakravarthy and Mandal [2]) studied effects of overlapping stenosis on arterial flow problem analytically by assuming the pressure variation only along the axis of tube. (Layek et al [8]) investigated the effects of overlapping stenosis on flow characteristics considering the pressure variation in both the radial and axial directions of the arterial segment under consideration and (Srivastava et al. [12]) studies the blood flow through an overlapping stenoses assuming that the flowing blood is represented by two layered macroscopic two-phase model . **(**Misra et al.[9]) developed a Herschel-Bulkley fluid model and observed that the resistance to flow and skin friction increase as stenosis height increases. (Shah[11]) studied the effect of Non-Newtonian behavior of blood flow through a radially non-symmetric multiple stenosis artery using Herschel-Bulkley fluid model and provided the results for the resistance to flow, apparent viscosity and the wall shear stress through graphical representations.

The effects of peripheral layer viscosity on physiological characteristics of blood flow through the artery with mild stenosis studied by (Shukla et al. [13]). It has been shown that the resistance to flow and wall shear stress decrease as the peripheral layer viscosity decrease. The effects of stenosis on resistance to flow and wall shear stress in an artery by considering the blood as non-Newtonian fluid showed by (Shukla et al. [14]). (Gupta.S et al. [6]) investigated the effects of stenosis and radial variation of viscosity on flow characteristics of blood considering laminar, incompressible and Non-Newtonian flow of blood using Power law fluid model. **(**Jain.N et al. [7]) observed various flow characteristics of blood and effect of parameters of stenosis using Herschel-Bulkley Non-Newtonian fluid model considering steady, laminar, one dimensional flow of blood through an axially non-symmetric but the radially symmetric atherosclerotic artery.

An attempt is made in the present investigation to explore the effect of radial variation of viscosity on the blood flow through an overlapping stenosis treating blood as Herschel-Bulkley fluid.

II. MATHEMATICAL FORMULATION

Consider the axisymmetric laminar and incompressible, steady, fully-developed, one dimensional flow of blood through a circular cylindrical tube under a constant pressure gradient with an overlapping constriction specified at the position as shown in Fig. 1. In this analysis, it is assumed that the stenosis developed in the arterial wall in an axially symmetric depends upon the axial distance z and the height of its growth. There is no external force acting on the flowing blood. Also, viscosity of blood varies along the radial direction and there exists a radial decrease in blood viscosity i.e., it is maximum at the axis of the artery and minimum near the wall. The geometry of the stenosis which is assumed to be manifested in the arterial segment is described Chakravarty and Mandal[2] as

$$\frac{R(z)}{R_0} = 1 - \frac{3}{2}\frac{\delta}{R_0 l_0^4}\left[11(z-d)l_0^3 - 47(z-d)^2 l_0^2 + 72(z-d)^3 l_0 - 36(z-d)^4\right], \quad d \le z \le d + l_0 \qquad \ldots(1)$$

$$= 1 \qquad , \text{otherwise} \qquad \ldots(2)$$

where R(z) and R_0 are the radius of the tube with and without stenosis, respectively, R_p is the radius of the plug flow region, l_0 is the length of the stenosis and d indicates its location, δ is the maximum projection (maximum height) of the stenosis into the lumen, appears at two locations: $z = d + \frac{1}{6}l_0$ and $z = d + \frac{5}{6}l_0$.The stenosis height at $z = d + \frac{1}{2}l_0$ from origin, called critical height, is $\frac{3\delta}{4}$.

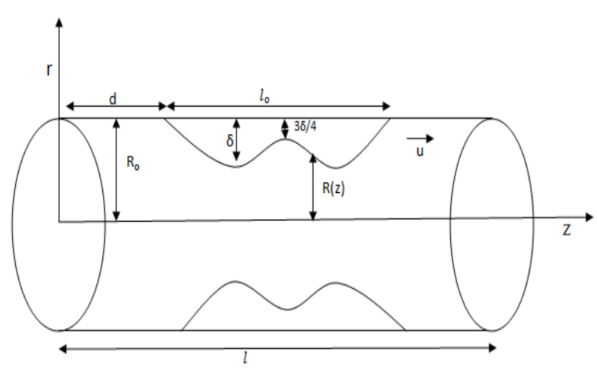

Fig. 1: The flow geometry of an arterial overlapping stenosis

The Navier-Stokes equation is given by

$$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\tau)=0 \qquad ...(3)$$

where r and z be the radial and axial coordinates respectively, p is the pressure and τ be the shear stress.

The constitutive equation for Herschel-Bulkley fluid is given by

$$\tau=\mu(r)\left(-\frac{\partial u}{\partial r}\right)^{n}+\tau_0 \ , \ \tau\geq\tau_0 \qquad ...(4)$$

$$\frac{\partial u}{\partial r}=0 \qquad , \ \tau<\tau_0 \qquad ....(5)$$

where τ_0 be the yield stress, $\mu(r)$ be the viscosity coefficient of blood and $\frac{\partial u}{\partial r}$ be the shear rate.

The boundary conditions pertaining to the problem are :

$u=0$ at $r=R(z)$ and ...(6)

τ is finite at $r=0$ ...(7)

The viscosity variation along the radial direction is linear and is as follows:

$$\mu(r)=\mu_0(1-q\frac{r}{R_0}) \qquad ....(8)$$

where μ_0 is the viscosity of the fluid at r = 0 and q(<<1) is a constant parameter.

III. ANALYTICAL SOLUTION

The equation of motion describing one dimensional flow of blood treating it as Herschel-Bulkley fluid is

$$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left\{\mu(r)\left(-\frac{\partial u}{\partial r}\right)^{n}+\tau_0\right\}\right]=0 \qquad ....(9)$$

To obtain the expression for velocity profiles, i.e., integrating equation (9) w.r.t. r on both sides under boundary condition (7)

$$u=\frac{-1}{2^{1/n}}\int\left(\frac{r\frac{\partial p}{\partial z}-2\tau_0}{\mu(r)}\right)^{1/n}dr \qquad ....(10)$$

The **flux** Q is given by

$$Q=\int_0^{R(z)}2\pi ru\,dr \qquad .....(11)$$

Now, substituting equations (10) and (8) in (11), we obtain

$$Q=\frac{\pi}{(2\mu_0)^{1/n}}\int_0^{R(z)}r^2\left\{\frac{r\frac{\partial p}{\partial z}-2\tau_0}{\left(1-q\frac{r}{R_0}\right)}\right\}^{1/n}dr \qquad ....(12)$$

Equation (12) can be restated as

$$\frac{\partial p}{\partial z}=2\mu_0\left(\frac{Q}{\pi I_{R(z)}}\right)^{n} \qquad .....(13)$$

Where $I_{R(z)}=\int_0^{R(z)}r^2\left\{\frac{r-\frac{2\tau_0}{\frac{\partial p}{\partial z}}}{\left(1-q\frac{r}{R_0}\right)}\right\}^{1/n}dr$

....(14)

To obtain **pressure drop** (Δp), we use the following conditions :

$p=p_0$ at $z=0$ and .....(15)

$p=p_L$ at $z=L$ .....(16)

Integrating equation (13) w.r.t z on both sides and applying boundary conditions (15) and (16), we obtain

$$\Delta p=p_l-p_0=\int_0^l\frac{\partial p}{\partial z}dz=2\mu_0\left(\frac{Q}{\pi}\right)^{n}\int_0^l\left(\frac{1}{I_{R(z)}}\right)^{n}dz \qquad ...(17)$$

Resistance to flow λ is defined as the ratio of pressure drop to the volumetric flow rate or flux. i.e.,

$$\lambda=\frac{\Delta p}{Q} \qquad ....(18)$$

Using equation (17), it can be written as

$$\lambda=2\mu_0\frac{Q^{n-1}}{\pi^n}\int_0^l\left(\frac{1}{I_{R(z)}}\right)^{n}dz \qquad ....(19)$$

For non-stenotic region, i.e. $(R(z)=R_0)$, resistance to flow is given by

$$\lambda_N=2\mu_0\frac{Q^{n-1}}{\pi^n}\int_0^l\left(\frac{1}{I_{R_0}}\right)^{n}dz \qquad ....(20)$$

Where $I_{R_0} = \int_0^{R_0} r^2 \left\{\frac{r-\frac{2\tau_0}{\frac{\partial p}{\partial z}}}{\left(1-q\frac{r}{R_0}\right)}\right\}^{1/n} dr$ ….(21)

So, the ratio of resistance to flow is obtained by

$$\lambda' = \frac{\lambda}{\lambda_N} \quad ….(22)$$

i.e., from equations (19) and (20), we have

$$\lambda' = \frac{\int_0^l \left(\frac{1}{I_{R(z)}}\right)^n dz}{\int_0^l \left(\frac{1}{I_{R_0}}\right)^n dz} \quad …..(23)$$

The **wall shear stress** is given by

$$\tau_w = \mu(r)\left(-\frac{\partial u}{\partial r}\right)^n + \tau_0\Big|_{r=R(z)} \quad ….(24)$$

Using equations (10) and (24), we have

$$\tau_w = R(z)\mu_0\left(\frac{Q}{\pi I_{R(z)}}\right)^n \quad …(25)$$

Now, at maximum stenosis height, the wall shear stress from equation (25) is given as

$$\tau_{wl} = R(z)\mu_0\left(\frac{Q}{\pi I_{R(z)}}\right)^n\Big|_{z=d+\frac{l_0}{2}} \quad ….(26)$$

The wall shear stress for normal artery is given as

$$\tau_{Nl} = R_0\mu_0\left(\frac{Q}{\pi I_{R_0}}\right)^n \quad ….(27)$$

At the wall, the ratio of shearing stresses is

$$\tau' = \frac{\tau_{wl}}{\tau_{Nl}} \quad ….(28)$$

i.e., from equations (26) and (27)

$$\tau' = \frac{R(z)\left(\frac{1}{I_{R(z)}}\right)^n\Big|_{z=d+\frac{l_0}{2}}}{R_0\left(\frac{1}{I_{R_0}}\right)^n} \quad ….(29)$$

IV. RESULTS AND DISCUSSION

The analytical expressions for volumetric flow rate, resistance to flow and the wall shear stress have been derived in the previous section.

The profile of Q given by equation (12) with axial distance to radius ratio $\frac{z}{R_0}$ for linearly radial variation of viscosity of fluid are plotted in Figs. 2 – 5, respectively, for distinct values of q, n, pressure gradient (p) and yield stress (τ_0). It can be easily seen from Fig. 2 that flow rate Q increases as q increases for fixed values of n and $\frac{z}{R_0}$. In Fig. 3, flow rate Q increases more rapidly for n=1(Newtonian) in comparison to n=2/3 or 1/3 (Non-Newtonian) for fixed values of $\frac{z}{R_0}$. Furthermore, from Figs. 4 and 5 it has been observed that the flow rate Q becomes higher for increasing values of pressure gradients and becomes lower for increasing values of yield stress.

The profile of Q given by equation (12), λ' given by equation (23) and τ' given by equation (29) with stenosis height to radius ratio $\frac{\delta}{R_0}$ for linearly radial variation of viscosity of fluid are plotted in Figs. 6 – 17, respectively, for distinct values of q, n, pressure gradient (p) and yield stress (τ_0).

It is clear from the Figs. 6, 10 and 14 that Q, λ', τ' increases as q increases for fixed values of n and $\frac{\delta}{R_0}$. In Figs. 11 and 15, λ' *and* τ' increases more rapidly for n=1(Newtonian) in comparison to n=2/3 or n=1/3 (Non-Newtonian) whereas in Fig. 7 Q decreases more rapidly for n=1(Newtonian) in comparison to n=2/3 or 1/3 (Non-Newtonian).

It can be noted from Figs. 12 and 16 that λ' *and* τ' increases as pressure gradient decreases for fixed value of $\frac{\delta}{R_0}$. Also from Fig. 8, flow rate Q becomes higher for increasing values of pressure gradient and decreases more rapidly for high values of pressure gradient as $\frac{\delta}{R_0}$ increases.

Furthermore, from Figs. 13 and 17, λ' *and* τ' increases as yield stress increases for fixed values of $\frac{\delta}{R_0}$. They both increases more rapidly for $\tau_0 = 0.05$ in comparison to $\tau_0 = 0$ or $\tau_0 = 0.02$ as $\frac{\delta}{R_0}$

increases. Also from Fig. 9, flow rate Q becomes higher for decreasing values of yield stress and decreases more rapidly for low values of yield stress as $\frac{\delta}{R_0}$ increases.

Finally, it can noted from Figs. 6 – 17 that λ' *and* τ' increases whereas Q decreases as $\frac{\delta}{R_0}$ increases.

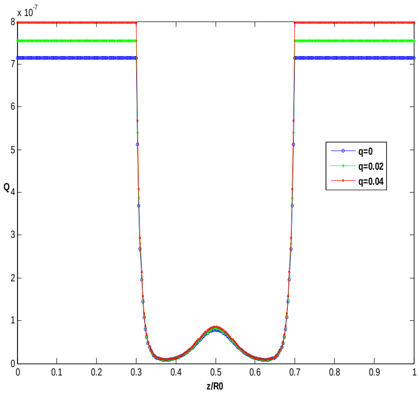

Fig. 2 : Profiles for Q against $\frac{z}{R_0}$ for distinct q

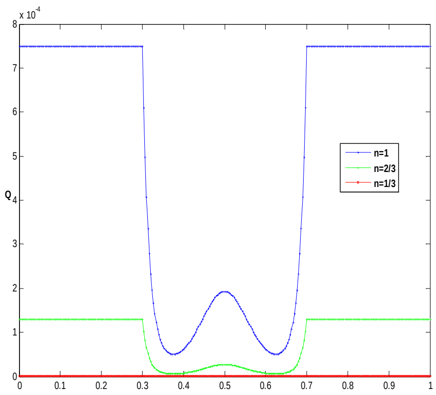

Fig. 3 : Profiles for Q against $\frac{z}{R_0}$ for distinct n

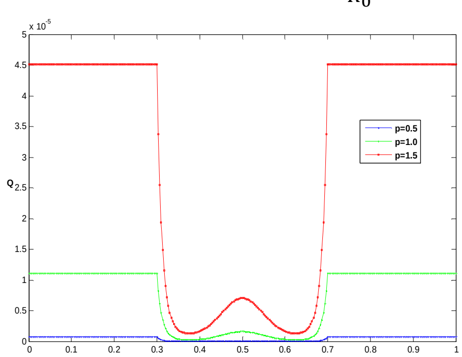

Fig. 4 : Profiles for Q against $\frac{z}{R_0}$ for distinct p

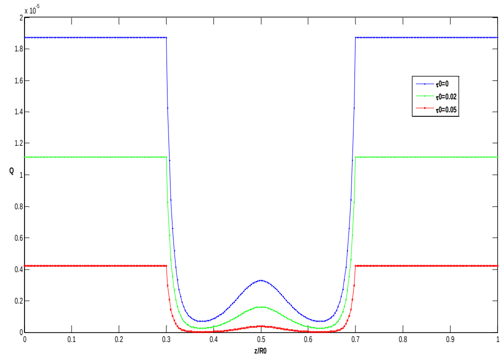

Fig. 5 : Profiles for Q against $\frac{z}{R_0}$ for distinct τ_0

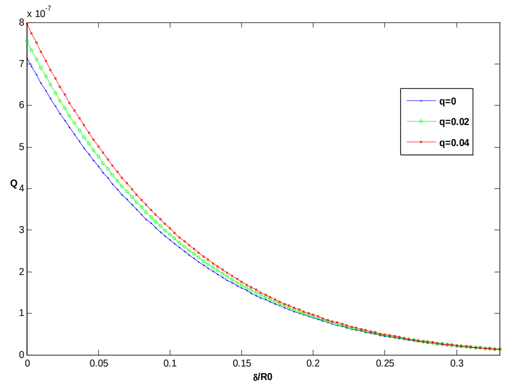

Fig. 6 : Profiles for Q against $\frac{\delta}{R_0}$ for distinct q

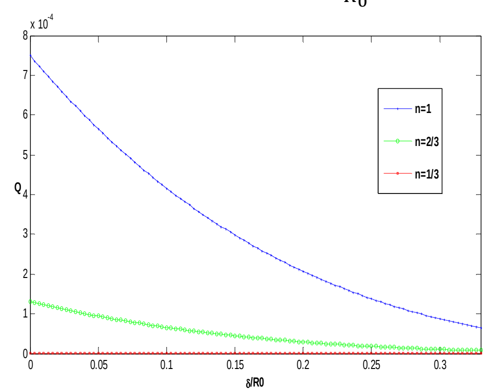

Fig. 7 : Profiles for Q against $\frac{\delta}{R_0}$ for distinct n

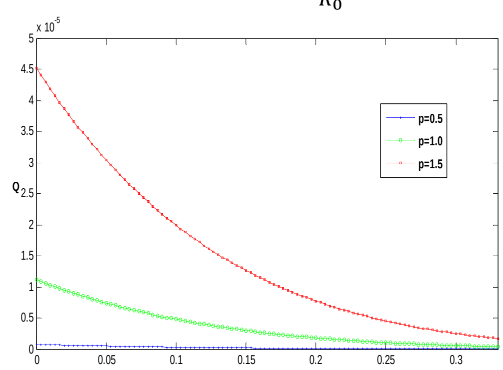

Fig. 8 : Profiles for Q against $\frac{\delta}{R_0}$ for distinct p

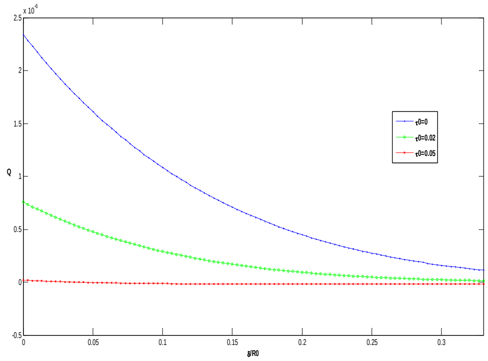

Fig. 9 : Profiles for Q against $\frac{\delta}{R_0}$ for distinct τ_0

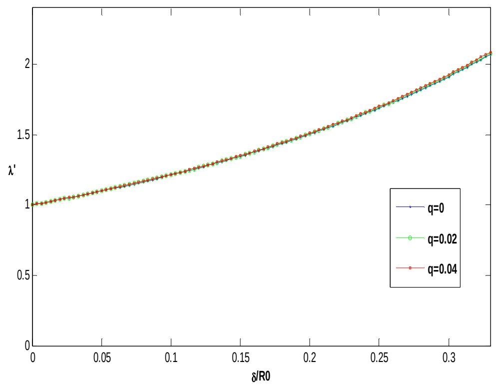

Fig. 10 : Profiles for λ' against $\frac{\delta}{R_0}$ for distinct q

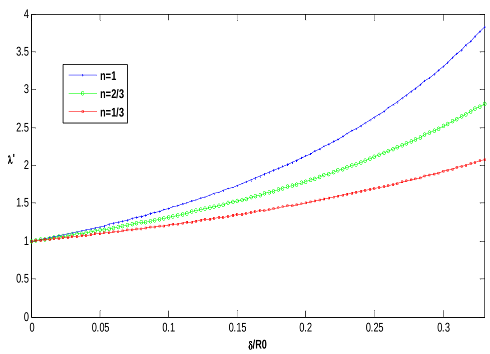

Fig. 11 : Profiles for λ' against $\frac{\delta}{R_0}$ for distinct n

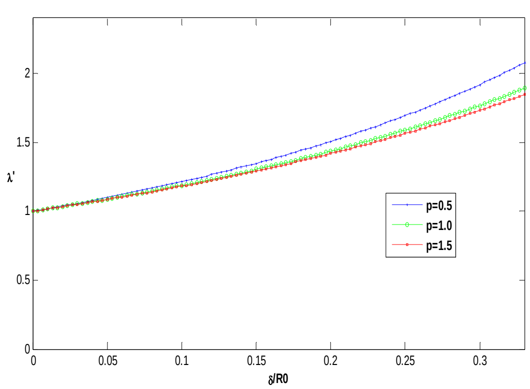

Fig. 12 : Profiles for λ' against $\frac{\delta}{R_0}$ for distinct p

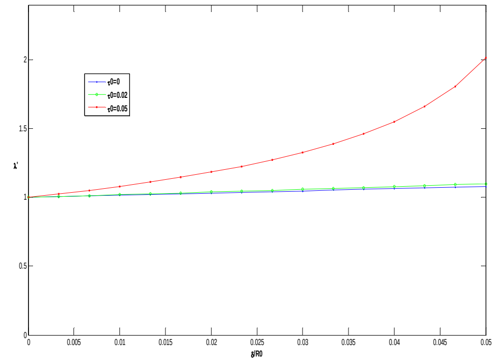

Fig. 13 : Profiles for λ' against $\frac{\delta}{R_0}$ for distinct τ_0

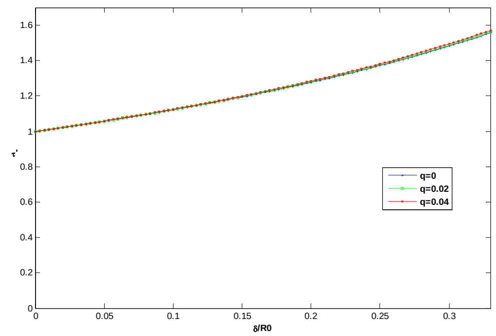

Fig. 14 : Profiles for τ' against $\frac{\delta}{R_0}$ for distinct q

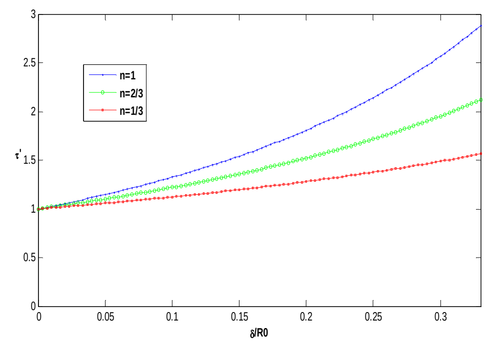

Fig. 15 : Profiles for τ' against $\frac{\delta}{R_0}$ for distinct n

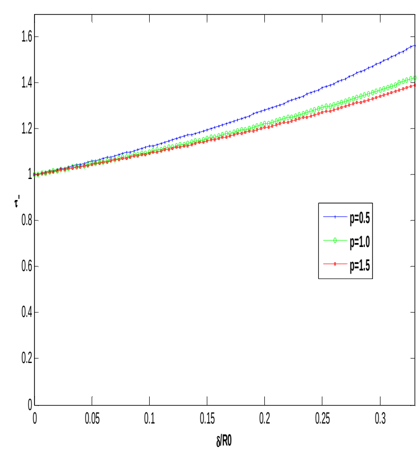

Fig. 16 : Profiles for τ' against $\frac{\delta}{R_0}$ for distinct p

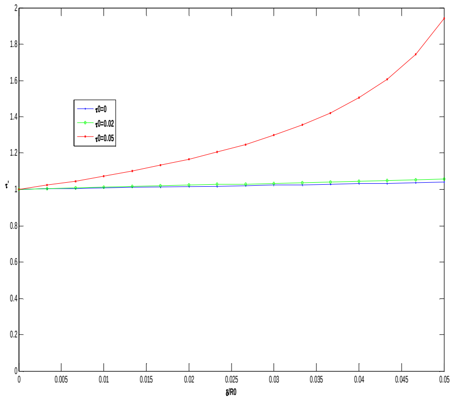

Fig. 17 : Profiles for τ' against $\frac{\delta}{R_0}$ for distinct τ_0

V. CONCLUSION

The analytical expression for the flow rate, resistance to flow and wall shear stress is obtained and results are discusses graphically. It is found that flow rate decreases as the height of stenosis is increased. Furthermore, resistance to flow and wall shear stress increases as the height of stenosis is increased. Flow rate of the fluid first decreases as the axial distance increases and then it increases with the value of axial distance and attains its minimum value when stenosis size is maximum within the stenosis region. This study may lead to some important results for clinical point of view.

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